A single photon produces general W state of N qubits and its application

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Based on the Wu's scheme[1], We prepare the general N-qubit W state. We find that the concurrence of two qubits in general N-qubit W state is only related to their coefficients and we successfully apply the general N-qubit W state to quantum state transfer and quantum state prepare like that in two-qubit system.

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I. INTRODUCTION

Quantum entanglement plays an important role in quantum information theory (QIT). It is regarded as an important resource in various kinds of quantum information processing and quantum computation[2]. Recently, there has been characterizing the entanglement properties of multi-particle systems. Several parties being spatially separated from each other and sharing a composite system of an entangled state buildup a situation of particular which is interest in QIT. This setting requires communication through a classical channel and only act locally on their subsystems. Although, the parties can still modify the entanglement properties of the system and try to convert one entangled state into another, even being restricted to local operations assisted with classical communication[3].

The definition of W state was firstly given by W. Dür. G. Vidal. and J. I. Cirac[4]. In that paper, W state consists of three qubits, and the coefficients are the same,

$$|W\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$$

Here, we give a general W state of N-qubit that is defined as

$$|W\rangle = \sum_i c_i|i\rangle, |i\rangle = |0_1 0_2 \ldots 1_i \ldots 0_N\rangle$$

where$|c_1|^2 + |c_2|^2 + |c_3|^2 + \ldots + |c_N|^2 = 1$,not all of the c_i are equalities. In Sec.II, we find that the concurrence of arbitrary m,n qubit in this state is only related to $|c_m|^2$ and $|c_n|^2$, and we give a new form of total concurrence of general N-qubit W state. By contrasting quantum state transfer and quantum state prepare in two-qubit system and general N-qubit W state, It suggests that we could transmit the quantum information by general W state as the same as that in two particles system.

In Ref.[1], the device gives us a way to generate W state, and by this way, we can prepare the general N-qubit W state. In that device, the teleportation is accomplished with mode entanglement. Mode entanglement recently becomes more and more useful in the quantum state transmit(QST)[5],[6],[7].

But for a single photon mode, we couldn't measure the photon to make sure its state if no capturing the photon. This leads that we have no way to realize the quantum information transmission. So we make some change about the device in order to capture the photon and finish the teleportation.

II. CONCURRENCE OF GENERAL N-QUBIT W STATE

Usually, concurrence is widely used to measure pairwise entanglement defined in spin-1/2 systems[8], it is the measurement of the two qubits entanglement[9],[10],[11]. The larger of concurrence, the more powerful of the entanglement is to be. Two qubits concurrence computation is given in Ref.[12], ρ_{ab} is the density matrix of system. Then we give the $\widetilde{\rho}_{ab}$:

$$\widetilde{\rho}_{ab} = (\sigma_y \otimes \sigma_y)\rho^*_{ab}(\sigma_y \otimes \sigma_y) \tag{1}$$

We define $R = \rho_{ab}\widetilde{\rho}_{ab}$, and the eigenvalues of R that are $\lambda_i, i = 1, 2, 3, 4$.

As given in Ref.[12], the concurrence of the system is

$$\begin{aligned} C_{ab} &= \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\} \\ &\quad \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4 \end{aligned} \tag{2}$$

Here, we calculate the concurrence of general N-qubit W state. The density matrix of general W state is given as follows.

$$\rho = |W\rangle\langle W| = \sum_{i,j} c_i^* c_j |j\rangle\langle i| \tag{3}$$

$|i\rangle = |0_1 0_2 \ldots 1_i \ldots 0_N\rangle$.

For getting the density matrix of the m-th and the n-th qubit, we take the trace of other qubits except the m-th,n-th qubit.

$$\begin{aligned} \rho_{m,n} &= \sum_{\acute{k} \neq m,n} \langle \acute{k} |\, \rho \,| \acute{k} \rangle \\ &= \begin{pmatrix} 1 - |c_m|^2 - |c_n|^2 & 0 & 0 & 0 \\ 0 & |c_n|^2 & c_m^* c_n & 0 \\ 0 & c_n^* c_m & |c_m|^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned} \tag{4}$$

where the basis $|k\rangle = |0_1 0_2 \ldots 0_{n-1} 0_{n+1} \ldots 0_{k-1} 1_k 0_{k+1} \ldots 0_{m-1} 0_{m+1} \ldots 0_N\rangle$

Easily, we get the eigenvalues of $R_{m,n}$, and from equ.3, we have the concurrence between m and n qubit.

$$C_{m,n} = 2|c_m||c_n| \tag{5}$$

And as the similar form to it in Ref.[12] the total concurrence of general N-qubit W state is

$$C_{total} = \sqrt{\frac{1}{2}\sum_{n\neq m} |C_{m,n}|^2} = \sqrt{2\sum_{n\neq m} |c_m|^2 |c_n|^2}$$

For this form, we find that when $|c_1| = |c_2| = \ldots = |c_N| = \frac{1}{\sqrt{N}}$, total concurrence reaches its maximum and $C_{total_{\max}} \geq 1$. But we hope the maximal total concurrence to be 1. So we give a new form of total concurrence:

$$C_{total} = \frac{\sqrt{2\sum_{n\neq m} |c_m|^2 |c_n|^2}}{\sqrt{2(1-\frac{1}{N})}} = \sqrt{(\frac{N}{N-1})\sum_{n\neq m} |c_m|^2 |c_n|^2} \tag{6}$$

where N is the number of qubit, $C_{total} \leq 1$. And when $N = 2, |c_1|^2 = |c_2|^2$, we will easily find the concurrence of ours is the same as Wootters concurrence.

As mentioned in Ref.[5], we write another form of total concurrence by using equ.5.

$$C_{total} = \sum_{j}^{N/2} C_{j,N+1-j} = 2\sum_{j}^{N/2} |c_j||c_{N+1-j}| \tag{7}$$

or

$$C_{total} = 2\sum_{j}^{(N+1)/2} |c_j||c_{N+1-j}| \tag{8}$$

For equ.7, N is even, and N is odd number in equ.8.

In Ref.[5], quantum state transfer is described as a dynamic process that starts from a initially factorized state to a factorized finally state though an entanglement state in the intermediate process. In that paper, when the fidelity reach maximum, the total concurrence approximately vanishes. And the maximal concurrence is 1. Here, we try to find the relationship between two kinds of concurrence. In our paper, $C_{total} = 0$, when $|c_i| \neq 0$ and $|c_{k\neq i}| = 0$ or $C_{j,N+1-j} = 0$. When $|c_i| \neq 0$ and $|c_{k\neq i}| = 0$, it means that we only have one state, necessarily the fidelity is 1. If $C_{j,N+1-j} = 0$, the system could not accomplish quantum state transfer. In Ref.[5], $C(t) = \sum C_{j,N+1-j}$, if $C_{j,N+1-j} = 0$, they couldn´t take the mode entanglement as the intermediate state. So the processing of QST is transfer quantum state from $|1\rangle$ to $|N\rangle$ through the middle state $|\psi(t)\rangle = U(t)|1\rangle$. It also finishes the QST by one state. Obviously, we have the same result. Then we find that when $|c_1| = |c_2| = \ldots = |c_N| = \frac{1}{\sqrt{N}}$, we receive $C_{total_{\max}} = 1$. Therefor, we get the same result again. Let´s review that paper, it suggests that $C(t)$ only refers to the pair qubits of mirror symmetry, the form is suitable for N as even. It implies that $C(t)$ is a especial kind of C_{total}. Whereupon, we explain the processing of QST in Ref.[5] by our way. Because of more generalization, we could consider the mode mentioned in Ref.[5] as a especial condition or a part of ours.

III. THE APPLICATION OF GENERAL N-QUBIT W STATE

From Ref.[12], we know that for a two-qubit system $|\psi\rangle = c_1|01\rangle + c_2|10\rangle$, we have

$$C_{total} = 2|c_1||c_2| \tag{9}$$

There is surprisingly similarity between equ.5 and equ.9. So when Alice wants to transfer quantum state from her qubit to Bob´s qubit, if they do not know their qubits in a two-qubit system or in the general N-qubit W state, we want to know whether there will be some difference. First, Alice transferes a unknown quantum state $|\varphi(0)\rangle = \alpha|0\rangle + \beta|1\rangle$ with two different system. In two-qubit system $|\psi\rangle = c_1|01\rangle + c_2|10\rangle$, after Bell combined-measurement, subsidiary particle introduction and unitary operation, we find that when $c_1 > c_2$, we have the probability which is $2|c_2|^2$, oppositely, the probability is $2|c_1|^2$, only as $c_1 = c_2$, we get the maximal probability.

In general N-qubit W state

$$|W\rangle = c_1|00\cdots 1\rangle + c_2|0\cdots 10\rangle + \cdots + c_N|10\cdots 00\rangle$$

The whole state is

$$\begin{aligned} |\Phi\rangle &= |\varphi(0)\rangle|W\rangle \\ &= c_1\alpha|000\cdots 1\rangle + c_1\beta|100\cdots 1\rangle \\ &\quad + c_2\alpha|00\cdots 10\rangle + c_2\beta|10\cdots 10\rangle + \cdots \\ &\quad + c_N\alpha|010\cdots 00\rangle + c_N\beta|110\cdots 00\rangle \end{aligned} \tag{10}$$

Alice operates the C-not on the first two qubits, she obtains that:

$$c_1\alpha|000\cdots 1\rangle + c_1\beta|110\cdots 1\rangle + c_2\alpha|00\cdots 10\rangle + c_2\beta|110\cdots 10\rangle + \cdots + c_N\alpha|010\cdots 00\rangle + c_N\beta|100\cdots 00\rangle$$

Then she does the Hadmard operation on the first

qubit, the result is

$$\frac{1}{\sqrt{2}}|00\rangle(c_1\alpha|00\cdots1\rangle+c_2\alpha|00\cdots10\rangle$$
$$+c_2\alpha|00\cdots100\rangle+\cdots+c_N\beta|00\cdots00\rangle)+$$
$$\frac{1}{\sqrt{2}}|10\rangle(c_1\alpha|00\cdots1\rangle+c_2\alpha|00\cdots10\rangle$$
$$+c_2\alpha|00\cdots100\rangle+\cdots-c_N\beta|00\cdots00\rangle)+$$
$$\frac{1}{\sqrt{2}}|01\rangle(c_1\beta|00\cdots1\rangle+c_2\beta|00\cdots10\rangle$$
$$+c_2\beta|00\cdots100\rangle+\cdots+c_N\alpha|00\cdots00\rangle)+$$
$$\frac{1}{\sqrt{2}}|11\rangle(c_N\alpha|00\cdots00\rangle-c_1\beta|00\cdots1\rangle$$
$$-c_2\beta|00\cdots10\rangle-c_{N-1}\beta|10\cdots000\rangle)$$

We take $\frac{1}{\sqrt{2}}|00\rangle(c_1\alpha|00\cdots1\rangle + c_2\alpha|00\cdots10\rangle + c_2\alpha|00\cdots100\rangle+\cdots+c_N\beta|00\cdots00\rangle)$ for example.

For $c_1\alpha|00\cdots1\rangle+c_2\alpha|00\cdots10\rangle+c_2\alpha|00\cdots100\rangle+\cdots+c_N\beta|00\cdots00\rangle$, after measuring by $\langle 0_30_4\cdots0_{N-1}|$, Bob has $|c_1|^2+|c_N|^2$ probability to get the $|\varphi(0)=\alpha|0\rangle+\beta|1\rangle$. He gets the total rate is $\frac{1}{4}(|c_1|^2+|c_N|^2)$. He wants to get the maximal probability. So He works the $\hat{U}$ on the $c_1\alpha|00\cdots1\rangle+c_2\alpha|00\cdots10\rangle+c_2\alpha|00\cdots100\rangle+\cdots+c_N\beta|00\cdots00\rangle$. $\hat{U}$ is a $M\times M$ matrix, $M=2^{N-1}$. The elements of matrix of $\hat{U}$ is

$$\begin{aligned}\hat{U}_{2,2} &= t\ ,\ \hat{U}_{3,M}=t^* \\ \hat{U}_{3,2} &= -\sqrt{(1-|t|^2)}\ ,\ \hat{U}_{2,M}=\sqrt{(1-|t|^2)} \\ \hat{U}_{k,k} &= 1, k\neq 2,3,M,\ \hat{U}_{M,L}=1, L=2^{N-2}+1\end{aligned}$$

Other $\hat{U}_{i,j}=0$

$$\begin{aligned}t &= \frac{1}{2}\{1-(-1)^{\frac{3}{2}[\frac{|c_1^2-c_N^2|+(c_1^2-c_N^2)}{c_1^2-c_N^2}]}\}\frac{c_N}{c_1} \\ &+\frac{1}{2}\{1-(-1)^{\frac{3}{2}[\frac{|c_N^2-c_1^2|+(c_N^2-c_1^2)}{c_N^2-c_1^2}]}\}\frac{c_1}{c_N}\end{aligned}$$

Here, we calculate the processing of quantum state transfer from the first qubit to the last. By similar computation, the result suggest that we also could realize transfer from n qubit to m qubit in this way.

After operation using $\langle 0_30_4\cdots0_{N-1}|$, we find that when $c_1>c_N$, we have the probability of $2|c_N|^2$, oppositely, the probability is $2|c_1|^2$, only as $c_1=c_N$, we get the maximal probability. We generalize the computation to the W state like that mentioned in Ref.[4]. By analogy, we get the maximal probability that is $\frac{2}{N}$.

Above all, we find that there are no different results between two systems when they are used in quantum state transfer.

As the same process, we accomplish quantum state prepare in two systems, a known state$|\phi\rangle=\alpha|0\rangle+\beta|1\rangle$ α,β are real and we give a normal state $\beta|0\rangle-\alpha|1\rangle=|\psi\rangle$, then we express $|0\rangle$ and $|1\rangle$ with $|\phi\rangle$ and $|\psi\rangle$.

$$|0\rangle = \alpha|\phi\rangle+\beta|\psi\rangle \tag{11}$$
$$|1\rangle = -\alpha|\psi\rangle+\beta|\phi\rangle \tag{12}$$

For $|\psi\rangle=c_1|01\rangle+c_2|10\rangle$, we obtain

$$\begin{aligned}&c_1|01\rangle+c_2|10\rangle \\ =\ &(c_2\beta^2-c_1\alpha^2)|\phi\rangle_1|\psi\rangle_2+(c_1\beta^2-c_2\alpha^2)|\psi\rangle_1|\phi\rangle_2 \\ &+(c_1+c_2)\alpha\beta|\phi\rangle_1|\phi\rangle_2-(c_1+c_2)\alpha\beta|\psi\rangle_1|\psi\rangle_2\end{aligned} \tag{13}$$

when $c_1=-c_2$, we make the measurement by ${}_1\langle\phi|$ and then finish the quantum state prepare.

For $|W\rangle=c_1|00\cdots1\rangle+c_2|0\cdots10\rangle+\cdots+c_N|10\cdots00\rangle$. We replace $|0\rangle$ and $|1\rangle$ with $|\phi\rangle$ and $|\psi\rangle$ on the first and the last qubit. We have the equation that is

$$\begin{aligned}&c_1|00\cdots1\rangle+c_2|0\cdots10\rangle+\cdots+c_N|10\cdots00\rangle \\ =\ &|0_20_30_4\cdots0_{N-1}\rangle\{[c_N\beta^2-c_1\alpha^2]|\phi_1\rangle|\psi_N\rangle+ \\ &[c_1\beta^2-c_N\alpha^2]|\psi_1\rangle|\phi_N\rangle+(c_1+c_N)\beta\alpha|\phi_1\rangle|\phi_N\rangle \\ &-(c_1+c_N)\alpha\beta|\psi_1\rangle|\psi_N\rangle\}+c_2|1_20_30_4\cdots0_{N-1}\rangle \\ &(\alpha|\phi_1\rangle+\beta|\psi_1\rangle)(\alpha|\phi_N\rangle+\beta|\psi_N\rangle)+ \\ &c_3|0_21_30_4\cdots0_{N-1}\rangle(\alpha|\phi_1\rangle+\beta|\psi_1\rangle)(\alpha|\phi_N\rangle+\beta|\psi_N\rangle) \\ &+\cdots+c_{N-1}|0_20_30_4\cdots1_{N-1}\rangle(\alpha|\phi_1\rangle+\beta|\psi_1\rangle) \\ &(\alpha|\phi_N\rangle+\beta|\psi_N\rangle)\end{aligned} \tag{14}$$

After measuring the second qubit with $\langle 0_20_30_4\cdots0_{N-1}|$, we find that if $c_1=-c_N$, we could easily accomplish the quantum state prepare. And the probability is $2|c_1|^2$. Obviously, there are also the same between two systems in quantum state prepare.

Above results indicate that we could apply the general N-qubit W state to the quantum state transfer and quantum state prepare like that in two-qubit system, we could use the same way to get the similar result.

IV. A SCHEME TO PREPARE GENERAL N-QUBIT W STATE AND QST

The device given in Ref.[1] could generate W state that has the same probability. As Fig.1, We find that we could change the reflectivity of the each beam splitter to get the general W state. Here we use the same probability W state for convenience and no losing general.

There is a single photon to prepare W state by mode entanglement or accomplish quantum sate transfer in the schemes in Refs.[1],[13],[14].We agree that the prepare of W state is viable, but there are two difficulties in the state transfer. One is the question of photon settlement. Without staying photon, we will not measure on the photon to realize the quantum state transfer for too long distance of photon having escaped. The other question comes from mode entanglement. In single photon system, the being or not of photon consist of the mode entanglement.

Once we measure the photon, the W state will collapse to a single state. So we couldn't measure the qubit in this condition, then we are no way to transmit the quantum information. Therefor, we need try to make some change about the device in order to realize the measurement. In this view, we take some cavities that they can capture the photon instead of photon counting detectors. In this way, we could finish distribute these cavities like particles. The people who have the cavities don't know whether there is photon in them. The new W state is $|W\rangle = c_1|1_1 0_2 \ldots 0_N\rangle + c_2|0_1 1_2 \ldots 0_N\rangle + \ldots + c_N|0_1 0_2 \ldots 1_N\rangle$. Here $|1_i\rangle$ or $|0_i\rangle$indicates that there is a photon or not in the i position hollow. After distributed, put a particle, unknown its state, into the first hollow. Then we make the operation on the cavities like doing on the particles, and we will get the state in some probability. With this changed device, we realize a single photon to prepare general N qubits W state and transmit the quantum information.

V. CONCLUSION

We have obtained the general W state by the device as Fig1. In this mode, we calculate the concurrence of the general N-qubit W state. As the result, the $C_{m,n}$ is only related to $|c_m|$ and $|c_n|$, and give a new form of total concurrence of general N-qubit W state. Then by contrasting quantum state transfer and quantum state prepare in two-qubit system and general N-qubit W state, we find we will get the same result whichever system the two qubits that transfer quantum state from one to another in. At last we improve the device to give a scheme that realize quantum state transfer and quantum state prepare. In this paper, we make the mode entanglement be useful in the processing of QST.

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